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B.Sc. Maths. (Hon.) (1st semester)

(Regular/Re-Appear/Improvement)
Examination, 2022

Algebra

Paper Code : BHM-111

Time Allowed: 3 Hrs. **[Max Marks: 60]**

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after examination.

Note: Attempt **Five** questions in all. **Q.No. 9 is Compulsory.** All questions carry equal marks.

1. (a)	Solve the following system of equation using matrix method: x+y+z+6=0, x+2y+3z+14=0, x+4y+9z+36=0.	6
(b)	Express $A=\begin{bmatrix} 1 & 0 & 5 & 3 \\ -2 & 1 & 6 & 1 \\ 3 & 2 & 7 & 1 \\ 4 & -4 & -2 & 0 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.	6
2. (a)	Find the rank of the matrix $A=\begin{bmatrix} 9 & 0 & 2 & 3 \\ 0 & 1 & 5 & 6 \\ 4 & 5 & 3 & 0 \end{bmatrix}$ by reducing it to normal form.	6
(b)	Find the characteristics vectors of the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.	6
3. (a)	For what values of k, the equations $x+y+z=1, x+2y+4z=k, x+4y+10z=k^2$ have a solution and solve them completely in each case.	6
(b)	Prove that the matrix $A=\frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ is orthogonal and find its inverse.	6
4.	Reduce the quadratic form $x^2+y^2+4z^2-9t^2-2xy-4yz+6yt-6tx-12tz$ into canonical form over complex field and also find the equations of transformations.	12
5. (a)	Solve the equation $4x^4+8x^3+13x^2+2x+3=0$, it being given that the sum of two of its roots is zero.	6

(b)	Solve the equation $x^4 - 9x^2 + 4x + 12 = 0$, given that it has a multiple root.	6
6. (a)	Find the condition that the roots of the cubic $x^3 - px^2 + qx - r = 0$, may be in H.P. Hence or otherwise solve the equation $6x^3 - 11x^2 + 6x - 1 = 0$,	6
(b)	Find the equation of squared differences of the roots of the equation $x^3 - 7x + 6 = 0$.	6
7. (a)	Show that the roots of the cubic equation $x^3 - 12x + 8 = 0$ are $4\cos\frac{2\pi}{9}, 4\cos\frac{4\pi}{9}, 4\cos\frac{8\pi}{9}$.	6
(b)	Solve the bi-quadratic $x^4 - 4x^3 + 9x^2 - 12x + 18 = 0$ by resolving it into quadratic factors.	6
8. (a)	Solve $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ by Ferrari's method.	6
(b)	Show that the equation $x^7 + x^4 + 8x + k = 0$ has at least 4 imaginary roots for all values of k.	6

9. (a)	If A and B are symmetric matrices, prove that AB is symmetric if and only if AB=BA.	2
(b)	Prove that 0 is the latent root of a matrix if and only if A is singular.	2
(c)	Prove that transpose of a unitary matrix is unitary.	2
(d)	Define rank of a matrix.	2
(e)	Find an equation whose one root is 2-3i.	2
(f)	State Cayley Hamilton's theorem.	2

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