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B. Sc. (Pass Course) 3rd Semester  
(Regular/Re-Appear/Improvement)

Examination – December 2022

MATHEMATICS (STATICS)

Paper : 12BSM233

Time : Three hours / Maximum Marks : 40

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Section. Question No. 9 (Section V) is compulsory. Marks are shown against each question.

SECTION – I

1. (a) A string of length  $l$  is fastened to two points A, B at the same level and at a distance ' $a$ ' apart. A ring of weight  $W$  can slide on the string and a horizontal force  $P$  is applied to it such that it is in equilibrium vertically below B. Show that  $P = \frac{aw}{l}$ , and tension of the string is  $\frac{w(l^2 + a^2)}{2l^2}$ . 3.5

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- (b) A heavy uniform rod 6 m long, rests horizontally on two pegs which are 1.5 m apart. A weight of 20 kg suspended from one end or a weight of 8 kg suspended from the other end will just tilt the rod up. Find the weight of the rod and the distances of the pegs from the centre of the rod. 3.5
2. (a) Forces of magnitude  $P, 2P, -3P, 4P, 5P$  and  $9P$  act respectively along the sides taken in order of a regular hexagon. Prove that they reduce to a resultant force acting through the centre of the hexagon. Find the magnitude and direction of their resultant. 3.5
- (b) The moments of a force about points  $(0,0), (10,0), (0,5)$  are 184, -46 and 249 kg. m. Find where the force meets the axis of  $x$  and find its components parallel to the co-ordinate axes. 3.5

SECTION – II

3. (a) Two uniform rods AB, BC rigidly joined at B so that angle ABC is a right angle, hang freely in equilibrium from a fixed point A. The length of the rods are  $a$  and  $b$  and their weights are ' $aw$ ' and ' $bw$ '. Prove that if AB makes an angle  $\theta$  with the vertical, then  $\tan \theta = \frac{b^2}{a^2 + 2ab}$ . 3.5
- (b) Two equal heavy rods of weight  $W$  and length  $2a$  are freely hinged together and placed symmetrically over a smooth fixed sphere of radius  $r$ . Show that the inclination of each rod to the horizontal is given by  $r(\tan^3 \theta + \tan \theta) = a$ . 3.5

(2)

4. (a) Find the limits between a force must lie in order to keep a body in equilibrium on a rough inclined plane, when the force acts horizontally. 3.5
- (b) Find the centre of gravity of the arc of the cardioid  $r = a(1 + \cos \theta)$  lying above the initial line. 3.5

## SECTION – III

5. (a) The necessary and sufficient condition that a particle acted upon by a number of coplanar forces be in equilibrium, is that the sum of the virtual works done by the forces in any small virtual displacement, consistent with the geometrical conditions of the system is zero. 3.5
- (b) A heavy uniform rod of length  $2a$  rests with its ends in contact with two smooth inclined planes of inclination  $\alpha$  and  $\beta$  to the horizon. If  $\theta$  be the inclination of the rod to the horizon, prove by the principle of virtual work that  $\tan \theta = \frac{1}{2}(\cot \alpha + \cot \beta)$  <https://www.iguonline.com> 3.5
6. (a) Show that every given system of forces acting on a rigid body can be reduced to a wrench. 3.5
- (b) Two forces act, one along the line  $y = 0, z = 0$  and the other along the line  $x = 0, z = c$ . As the forces vary, show that the surface generated by the axis of their equivalent wrench is  $(x^2 + y^2)z = cy^2$ . 3.5

## SECTION – IV

7. (a) To show, that a given system of forces can be replaced by two forces, equivalent to

the given system, in an infinite number of ways and that the tetrahedron formed by the two forces is of constant volume. 3.5

- (b) Find the null point of the plane  $lx + my + nz = 1$  for the system of forces  $(X, Y, Z; L, M, N)$ . 3.5

8. (a) A heavy uniform cube balances on the highest point of a sphere whose radius is  $r$ . If the sphere is rough enough to prevent sliding and if the side of cube be  $\frac{\pi r}{2}$ , show that the cube can rock through a right angle without falling. 3.5
- (b) A heavy body, the section of which is a cycloid, rests on a rough horizontal plane and has its C. G. at the centre of curvature of the curve at the point of contact. Show that the equilibrium is unstable. 3.5

## SECTION – V

9. (a) State Lami's theorem. 2
- (b) State  $\lambda - \mu$  theorem. 2
- (c) Find the centre of gravity of a uniform parallelogram lamina. 2
- (d) Define friction and force of friction. 2
- (e) Prove that the virtual works done by the tension in a virtual extension of a string from length  $l$  to  $l + \delta l$  is  $-T \delta l$ , where  $T$  is the tension in the string. 2
- (f) Define stable and unstable equilibrium. 2