Roll No. .. ..

51181

B. Sc. (Pass Course) 5th Semester (Regular/Re-Appear/Improvement) (Mercy Chance) Examination – December-2023

MATHEMATICS - I (Real Analysis)

Paper : BSM501

Time : Three Hours] *[Maximum Marks : 40*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. **9 (Section – V)** is ***compulsory***. Marks are indicated against each question.

SECTION – I

1. (a) By using definition, find $U(f, P)$ and $L(f, P)$. Also evaluate $\int_0^k x^3 dx = \frac{k^4}{4}$. 3½

P. T. O.

(b) Show that : 3½

$$\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\ldots+\frac{1}{2n}\right]=\log_e 2$$

2. (a) Prove that every bounded monotonic function is an integrable function. 3½

(b) State and prove Fundamental Theorem of Integral Calculus. 3½

SECTION – II

3. (a) Examine the Convergence of integral : $\int_0^1 \frac{x^p \log x}{(1+x)^2}$. 3½

(b) Discuss the convergence of Gamma Function. 3½

4. (a) State and prove Frullani's integral for convergence. 3½

(b) Prove that $\int_0^{\frac{\pi}{2}} \log\left(\frac{\alpha+\beta\sin\theta}{\alpha-\beta\sin\theta}\right)\frac{d\theta}{\sin\theta} = \pi\sin^{-1}\frac{\beta}{\alpha}$, if $\alpha > \beta$. 3½

SECTION – III

5. (a) Let A, B be subsets of a metric space (X, d). Prove that $\delta(A \cup B) \le \delta(A) + \delta(B) + d(A, B)$. 3½

(b) Prove that A be a subset of metric space (X, d). Then A° is union of all open sets contained in A. 3½

6. (a) Prove that every convergent sequence in a metric space is a Cauchy Sequence but converse need not be true. 3½

(b) State and prove Cantor's Intersection theorem. 3½

SECTION – IV

7. (a) Let (X, d) and (Y, d^*) be two metric spaces. A function $f : X \rightarrow Y$ continuous at a point $a \in X$ iff for every sequence $< a_n >$ in X converging to a the sequence $< f(a_n) >$ in Y converges to $f(a)$. 3½

(b) A metric space (X, d) is compact if and only if it has Bolzano Weierstrass Property. 3½

8. (a) Prove that every compact (sequentially compact) metric space is complete. 3½

(b) A metric space (X, d) is disconnected iff there exists a non-empty proper subset of X which is both open and closed. 3½

SECTION – V

9. (a) Examine the convergence of improper integral : 2

$$\int_0^\infty \frac{1}{1+x^2}\, dx$$

(b) State Darboux's theorem. 2

(c) Let (X, d) be metric space and let A, B subsets of X. Then prove that $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$. 2

(d) Give an example of collection of subsets having finite intersection property. 2

(e) Show that every subset of a discrete metric space is closed. 2

(f) Prove that an isometry is uniformly continuous function. 2