Roll No. ....

22048

M. Sc. Mathematics 2nd Semester

(Regular/Re-Appear/Impr.)

Examination – May, 2024

MEASURE AND INTEGRATION THEORY

Paper : MAT202

Time : Three Hours] ***[Maximum Marks : 80***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** **questions** in all, selecting *one* question from each Section. Question No. 9 (Section - V) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) Show that union of two **measurable sets** is measurable. **6**

(b) Let A be an algebra of sets and $\{E_i\}_{i \in N}$ be a sequences of subsets in A then there exists a sequence $\{D_i\}_{i \in N}$ of disjoint members of A s. t.

$D_i \subseteq E_i \ \forall \ i \in N$ and $\bigcup_{i \in N} D_i = \bigcup_{i \in N} E_i$. 10

2. (a) Show that the collection of all measurable sets is σ-algebra. 8

(b) Define a ~~Bord~~ set and show that every Bord set is measurable. 8

SECTION – II

3. (a) If f is a function defined on a measurable set E. Then f is measurable iff any open set G in IR, $f^{-1}(G)$ is a measurable set. 6

(b) Show that a function is measurable iff both its +ve and -ve parts are measurable. 6

(c) Let E be a set of rational in [0, 1] ~~per~~ show that X_E is measurable. 4

4. (a) Prove the converse of Egoroff's theorem. 8

(b) Prove that almost uniform convergence implies convergence in measure. 8

SECTION – III

5. (a) State and prove Fatou's lemma and show where strict inequality occurs. 10

(b) If f is a non-negative function show $F(x) = \int_{-\infty}^{x} f(t)\,dt$ is continuous on IR. 6

6. (a) If f is integrable function s. t. f = 0 a.e. show ∫f = 0. 8

(b) State and prove Lebesgue Dominated convergences theorem. 8

SECTION – IV

7. (a) Show that every non-decreasing function defined on $[a, b]$ is of bounded variation. 8

(b) If f is of bounded variation then show that f is bounded. 4

(c) Show that a function of bounded variation is not necessarily cont. 4

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8. (a) If f and g are functions of bounded variation and λ is a const. then show that $\lambda \cdot f$ and $f + g$ are functions of bounded variation and 8

$$V_a^b(f + g) \leq V_a^b(f) + V_a^b(g)$$

$$V_a^b(\lambda f) \leq (\lambda) V_a^b(f)$$

(b) If f is almost continuous on $[a, b]$ then show that it is of bounded variation. 8

SECTION – V

9. (a) Show that [0, 1] is not countable. 2 × 8 = 16

(b) Define a measurable set.

(c) Define the property a.e.

(d) Give an example of a function which is measurable but not continuous.

(e) give an example of a function f s.t. $\int_E f = 0$ but $f \neq 0$ a.e.

(f) State Monotone convergence theorem.

(g) Define function of bounded variation.

(h) State Jensen's inequality.